Quasi-second order positivity preserving ERK method for inviscid flows with large time-steps

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In this work, we present a quasi-second order in space and time finite volumes scheme on staggered meshes for inviscid flows that preserves positivity under a large CFL condition, which allows for fast computations with large time-steps.

The time-stepping strategy is a positivity-preserving multi-stages Explicit Runge-Kutta (ERK) method from [1]. It relies on the use of a flux limiter to ensure positivity. The more the stages, the larger the CFL number can be chosen.

We apply it in the context of a finite volumes scheme for inviscid flows on staggered meshes. The limiter we use is the Algebraic-MUSCL (A-MUSCL) algorithm [2]. It computes quasi-second order positivity preserving inter-cell values by projecting high-order (not positivity preserving) inter-cell approximations on a stability interval given by the values of adjacent cells. It is an easy-to-implement limiter that works well regardless of the space dimension.

To ensure consistency with staggered meshes, we need a discrete mass balance equation on the dual mesh and to add corrective terms to ensure the discrete total energy balance is conservative. We achieve this by using the A-MUSCL limiter to transform the ERK stage update into a Euler-like update, for which these mechanisms are already known [3].

This work is the first step to make an IMEX Navier-Stokes solver that does not take diffusion into account at every stage of the ERK method, following the work from [4].

Keywords: Compressible Navier-Stokes equations ; Low Mach number flows ; Pressure correction schemes ; Staggered approximations ; Finite volumes.

References

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