An energy stable and well-balanced scheme for the Ripa system

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We design and analyse an energy-stable, structure-preserving, and well-balanced scheme for the Ripa system of shallow water equations. The energy stability of the numerical solutions is achieved by introducing appropriate stabilisation terms in the discretisation of the convective fluxes of mass and momentum, the pressure gradient, and the topography source term. The careful selection of the water height and temperature interface values ensures the scheme's wellbalanced property for three physically relevant hydrostatic steady states. The scheme, which is explicit in time and finite volume in space, preserves the positivity of both the water height and the temperature, and it is weakly consistent with the continuous model equations in the sense of Lax-Wendroff. Additionally, a suitable modification of the source term discretisation and timestep criterion allows the scheme to handle wet/dry fronts in equilibrium. The results of extensive numerical case studies on benchmark test problems confirm the theoretical findings.