Asymptotic preserving finite volume method for the compressible Euler equations: analysis via dissipative measure-valued solutions

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We propose and analyse a new asymptotic preserving (AP) finite volume scheme for the multidimensional compressible barotropic Euler equations to simulate low Mach number flows. The proposed scheme uses a stabilized upwind numerical flux, with the stabilization term proportional to the stiff pressure gradient, and we prove its conditional energy stability and consistency. Using the concept of dissipative measure-valued (DMV) solutions, we rigorously illustrate the AP properties of the scheme for well-prepared initial data. In particular, we prove that the numerical solutions will converge weakly to a DMV solution of the compressible Euler equations as the mesh parameter vanishes, while the Mach number is fixed. The DMV solutions then converge to a classical solution of the incompressible Euler system as the Mach number goes to zero. Conversely, if the mesh parameter is kept fixed, we obtain an energy-stable and consistent finite-volume scheme that approximates the incompressible Euler equations as the Mach number goes to zero. The numerical solutions generated by this scheme then converge weakly to a DMV solution of the incompressible Euler system as the mesh parameter vanishes. Invoking the weak-strong uniqueness principle, we conclude that the DMV solution and classical solution of the incompressible Euler system coincide, proving the AP property of the scheme. We also present an extensive numerical case study to illustrate the theoretical convergences, in which we utilize the techniques of $\mathcal{K}$-convergence.