Entropy stable high-order discontinuous Galerkin schemes for compressible flows on moving meshes

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Keywords: high order numerical methods, shock capturing, moving mesh methods.

ABSTRACT

The discontinuous Galerkin method allows a straightforward construction of efficient high-order methods on arbitrary grids for hyperbolic partial differential equations (PDEs) such as the Euler equations. However, hyperbolic PDEs admit discontinuities in the solution, even if the initial solution is smooth. Moreover, it is well known that high-order schemes induce oscillations at discontinuities, called Gibbs phenomena. Hence, adequate numerical treatment is necessary to detect and handle discontinuities, ranging from shock fitting to shock capturing approaches. This is of particularly importance in the context of moving meshes, where the shock capturing has to satisfy an additional criterion such as the geometric conservation law on the discrete level, in order to ensure conservation of the numerical scheme.

One example of a shock capturing procedure is an h-refined low-order finite volume subcell scheme [2]. This allows an accurate handling and sharp representation of discontinuous. However, the coupling of a low-order FV with a high-order DG operator is well-known to give rise to unphysical oscillations in the numerical scheme. This can be reduced by using instead of a switching of a troubled cell to FV, a convex blending of a low-order FV and a high-order DG method, cf. [1]. Conversely, for stronger discontinuities, a switching procedure tends to ensure a more stable and accurate shock capturing.

In this talk, we propose a shock capturing approach which combines the switching and blending procedure to enable an entropy stable shock capturing scheme for static and moving meshes. The mesh movement is thereby based on the arbitrary Lagrangian–Eulerian method. The freestream preservation, high-order convergence properties and shock capturing capabilities of the resulting scheme are demonstrated.

References

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