Advection-pressure splitting schemes applied to a non-conservative 1D blood flow model with transport for arteries and veins

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Abstract

We consider a one-dimensional (1D) blood flow model with discontinuous mechanical and geometrical parameters. This type of model becomes relevant in medical contexts where these attributes exhibit spatial variations. These changes can occur, for example, due to the insertion of stents in arteries or veins after a surgical procedure, with the aim of restoring the vessel lumen to its original shape.

We introduce new first order, splitting-based numerical schemes for the non-conservative 1D blood flow equations with a general constant momentum correction coefficient that describe blood flow, for different velocity profiles, in arteries and veins with discontinuous mechanical and geometrical properties. In this model an advection equation for a passive scalar transport is also considered.

Our schemes are inspired by the original flux-vector splitting approach of Toro and Vázquez-Cendón (2012) [1] designed for the Euler equations. They also represent an improvement of the work proposed by Toro et al. (2024) [2] regarding non-conservative blood flow models, which considered a tube law describing only arteries, a momentum correction coefficient equal to one, no passive scalar transport and included a smaller number of discontinuous mechanical and geometrical parameters. The considered framework separates advection terms and pressure terms, generating two different systems of PDEs: the advection system in conservative form, and the pressure system in non-conservative form, both of which have a very simple eigenstructure compared to that of the full system. Our schemes involve approximate Riemann solvers and present a modification of the path-conservative framework that renders unnecessary the use of a path. They are systematically assessed on a carefully designed suite of test problems with exact solution and compared with several existing mainstream methods. A detailed efficiency analysis is performed in order to showcase the advantages of the proposed methodology in comparison to standard approaches.

An analogue work has already been carried out for a conservative 1D blood flow model with transport [3]. This research work has been submitted in [4].

References

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