A meshfree arbitrary Lagrangian-Eulerian method for the Boltzmann BGK model with moving boundaries

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We present a numerical method for simulating rarefied gasses that interact with moving boundaries and rigid bodies. The rarefied gas is modelled by the BGK-Boltzmann equation written in Lagrangian form and the motion of the rigid bodies is governed by the Newton-Euler equations as explained in [6]. The equation is then solved using an Arbitrary Lagrangian-Eulerian method in which the grid points move with the local mean velocity of the gas [6]. The main advantage of the moving grid is that the algorithm can deal well with cases where the simulation domain contains rigid objects since no cut-cell technique [4] or immersed boundaries [3] are required. The BGK equations are solved using higher-order Implicit-Explicit (IMEX) Runge-Kutta methods in which the advection term is solved explicitly, and the stiff relaxation terms implicitly. The main novelty of our approach is the use of a meshless MUSCL-like spatial discretization, in which the states are reconstructed at the midpoints using a Moving Least Squares (MLS) method. Since only a first-order MLS method can guarantee the maximum principle, we make use of a Multi-dimensional Optimal Order Detection (MOOD) scheme [2] to avoid spurious oscillations due to the transport step. We compare our approach to a WENO MLS method based on [1] that has been used in [6]. In all test cases, we consider diffusive reflection boundary conditions. The zero mass flux condition is applied immediately after the transport step (and thus before the collision step). Therefore, our method does not require extrapolation to the boundaries or an iterative procedure as in [5].

To showcase the versatility of the algorithm, we apply it to several test cases in 1D and 2D. In 1D, we use the new second and fourth-order methods for Sod's shock tube and the moving slider problem. For Sod's shock tube, our methods are significantly less diffusive than the WENO MLS method. In 2D, we consider the driven cavity problem with moving objects inside the domain, and the shear layer problem; see figure 1 for some preliminary results. Both simulations are performed with an ARS222 IMEX scheme combined with a second-order meshless MUSCL scheme and MOOD on 32 CPUs. The combination of the MOOD with the versatile MLS scheme results in a promising algorithm for complex simulations in the field of rarefied gas dynamics.

REFERENCES

[1] Avesani, D., Dumbser, M., and Bellin, A. A new class of Moving-Least-Squares WENO–SPH schemes. *Journal of Computational Physics 270* (Aug. 2014), 278–299.

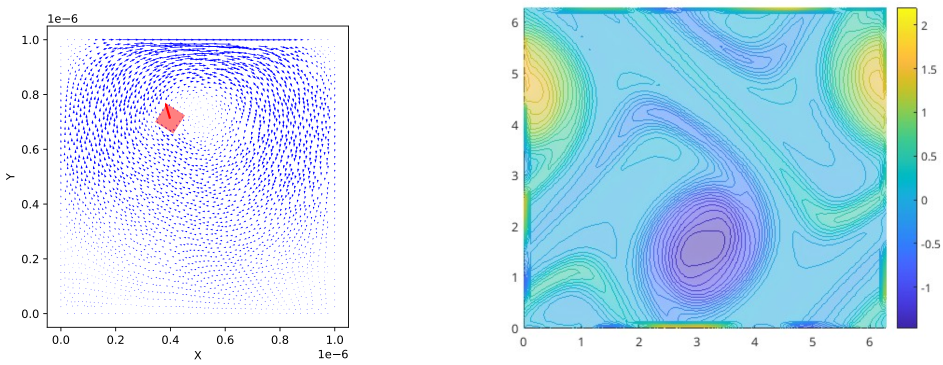
[2] Clain, S., Diot, S., and Loubère, R. A high-order finite volume method for systems of conservation laws—Multi-dimensional Optimal Order Detection (MOOD). *Journal of Computational Physics 230*, 10 (May 2011), 4028–4050.

[3] Dechristé, G., and Mieussens, L. Numerical simulation of micro flows with moving obstacles. *Journal of Physics: Conference Series 362* (May 2012), 012030.

[4] Dechristé, G., and Mieussens, L. A Cartesian cut cell method for rarefied flow simulations around moving obstacles. *Journal of Computational Physics 314* (June 2016), 465–488.

[5] Groppi, M., Russo, G., and Stracquadanio, G. Boundary conditions for semi-Lagrangian methods for the BGK model. *Communications in Applied and Industrial Mathematics 7*, 3 (Sept. 2016), 138–164.

[6] Tiwari, S., Klar, A., and Russo, G. A meshfree arbitrary Lagrangian-Eulerian method for the BGK model of the Boltzmann equation with moving boundaries. *Journal of Computational Physics 458* (June 2022), 111088.



(a) $Kn = 1.69 \times 10^{-3}$ **(b)** $Kn = (2\pi)^{-1} \times 10^{-6}$

Fig. 1. Left: A shear velocity is applied to the top boundary which creates a vortex inside a domain that contains a square rigid body. The velocity field of the gas is plotted. Right: A shear flow on a periodic domain with small initial perturbation generates a vortex. The vorticity is plotted.

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