Numerical entropy production in finite volume ADER schemes

M. Semplice, A. Zappa*

Abstract

Whenever we deal with conservation laws, uniqueness of weak solutions can be guaranteed by the entropy inequality. We are interested in the residual of this inequality, which represents the numerical entropy production by the approximation scheme we are considering. This idea has been introduced and exploited in Runge-Kutta finite volume methods, where the numerical entropy production has been used as an indicator in adaptive schemes, since it scales as the local truncation error of the method for smooth solutions and it highlights the presence of discontinuities and their kind.

The aim of this talk is to extend this idea to finite volume ADER timestepping techniques. We show that the numerical entropy production can be defined also in this context and it provides a scalar quantity computable for each space-time volume which, under grid refinement, decays to zero with the same rate of convergence of the scheme for smooth solutions, it's bounded on contact discontinuities and divergent on shock waves. We also present numerical evidence showing that it is essentially negative definite. Moreover, we propose an adaptive scheme that uses the numerical entropy production as smoothness indicator. The scheme locally modifies its order of convergence with the purpose of removing the oscillations due to the high-order of accuracy of the scheme.

References

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