Step-size selection and error control of the Runge-Kutta semi-Lagrangian scheme for the BGK equation

Bernardo Collufio (Gran Sasso Science Institute) and Giovanni Russo (University of Catania)

Abstract:

The BGK equation plays a crucial role in gas dynamics, approximating the effect of collisions near equilibrium. Over the years, several numerical methods have been developed to solve this equation. Among these, Eulerian methods, based on a direct discretization of the equation over a fixed grid, offer high accuracy and can preserve fundamental physical properties. However, they are constrained by the CFL condition, which may significantly limit efficiency.

Recently, semi-Lagrangian schemes have gained attention for their good accuracy and stability properties. These methods still use a Cartesian mesh but handle convection through Lagrangian formulation, integrating the system along characteristics. This approach results in unconditionally stable schemes, free from any a priori CFL restriction on the time step, which would otherwise severely degrade efficiency.

For such schemes, the time step cannot be decided based on stability considerations. Step size selection may be crucial for controlling local error, particularly in the presence of discontinuities (shocks), which occur for small Knudsen numbers, where inadequate time refinement can lead to rough solutions or inefficient schemes. The purpose of the talk is to present recent results on time step control for semi-Lagrangian schemes for the BGK model, an issue still quite unexplored in the literature on semi-Lagrangian schemes.