Compact implicit high resolution numerical method for solving transport problems with sorption isotherms

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This study investigates numerical methods for solving nonlinear transport problems characterized by various sorption isotherms [1] with a focus on the Freundlich type of isotherms [3]. We describe and compare second order accurate numerical schemes, focusing on compact implicit methods, to effectively model transport phenomena without stability restriction on the choice of time steps. To find the solution, the fast sweeping method is embraced in the work. Furthermore, a high resolution form of the method [2] is proposed to keep the values of numerical solutions in a physically acceptable range.

Through 1D and 2D numerical experiments, we demonstrate the effectiveness of high resolution methods in minimizing oscillations near discontinuities, thereby enhancing solution plausibility. The observed convergence rates confirm that the second order accurate schemes achieve expected accuracy for smooth solutions and that they yield significant improvements when compared with the results of the first order scheme. As the computational cost of the compact implicit method seems to be comparable to similar explicit ones with a clear profit of unconditional stability, this research provides a practical tool toward numerical simulations of nonlinear transport phenomena applicable in various fields such as contaminant transport in porous media or column liquid chromatography.

References

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