On the Entropy and Energy Conserving Properties of the Potential Temperature Formulation in the Euler Equations for Atmospheric Applications

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1 Abstract

In the last decade, there has been a growing interest in developing and studying new dynamical cores for climate and weather simulations based on the Discontinuous Galerkin (DG) approach. Various numerical methods and formulations of the Euler equations are employed to achieve accuracy, efficiency, and stability. In this work, we develop novel structure-preserving methods for the Euler equations, treating the potential temperature as a prognostic variable. We construct three numerical fluxes, ensuring entropy and total energy conservation, and extend the analysis to include a geopotential source term within the DG framework on general curvilinear meshes. Furthermore, we investigate a generalization of the kinetic energy-preserving and total energy-conserving properties when coupled with a generic geopotential term, along with well-balanced schemes for different constant background states. To achieve these properties, we adopt a flux-differencing approach for the discretization of the source term. Finally, we present several test cases to assess the theoretical findings and compare the potential temperature formulation to the traditional Euler equations formulation, on different classical atmopsheric test cases.