On the Active Flux scheme applied to a kinetic equation which is asymptotic-preserving

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The Active Flux (AF) method is a compact high-order finite volume method, where by adding point values at cell interfaces as additional degrees of freedom to the evolution of cell averages, this adds flexibility to the method. We use the method of lines for this scheme.

A common way for the point value update is based on Jacobian splitting (JS) method, incorporating the upwind idea. One of the advantages of the AF method over standard finite volume methods is its structure-preserving property: it preserves the vorticity and stationary states for the multi-dimensional acoustic equations, and it is naturally well balanced for acoustics with gravity.

These encouraging results lead us to the following question: Does the AF method also possess the asymptotic-preserving (AP) property in diffusive regime? To start, we study the hyperbolic heat equation, the lowest-order angular discretization of the transport equation, also known as P_1 model. We show that the JS-based AF method is AP for solving the hyperbolic hear equation, in the sense that the limit scheme is a discretization of the limit heat equation. We use formal analysis and numerical experiments to illustrate our findings.