Entropy-stable fluxes for nonconservative systems and their application for the Saint-Venant-Exner equations

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Ensuring entropy stability in numerical methods for hyperbolic conservation laws is crucial to achieve robust and physically consistent approximations. Such methods can be constructed by combining entropy-conservative fluxes with suitably designed entropy-stable dissipation. A key element in this construction is the inherent link between mathematical entropy and symmetrization. For conservative systems, the Hessian of the entropy function acts as a symmetrizer, which is commonly used to create entropy-stable dissipation operators. However, for nonconservative systems, even when an entropy function exists, these symmetry properties break down. This loss of structure has significant implications for entropy stability, making conventional entropy-stable flux formulations, that rely on this symmetrizer inapplicable. To address this challenge, we demonstrate entropy stability for a baseline dissipation using only convexity arguments and propose a novel blending procedure to construct different entropy-stable fluxes. We then apply this approach to obtain an entropy-stable discontinuous Galerkin spectral element method for the Saint-Venant-Exner system. Finally, we present computational results demonstrating the effectiveness of our blending approach compared to existing numerical fluxes.