Barotropic-Baroclinic Splitting for Multi-Layer Rotating Shallow Water Models

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The classical shallow water equations have a limited range of application as they cannot model vertical effects. The multi-layer approach ([1] and others) allows for capturing some vertical variations in the flow while still maintaining a simplification compared to the 3D Euler equations. Computational costs obviously increase with the number of layers, which is often around 50 in ocean simulations. The barotropic-baroclinic splitting is a strategy used in (numerical) ocean models [2] to reduce the computational costs.

In this contribution, we focus on the numerical analysis of the barotropic-baroclinic splitting in the context of Finite Volume schemes. We reformulate this splitting strategy within the nonlinear multi-layer framework using terrain-following coordinates and present it as an exact operator splitting. The barotropic step captures the evolution of the free surface and depth averaged velocity with a well-balanced one-layer shallow water model [3]. The baroclinic step incorporates vertical exchanges between layers and adjusts velocities around their mean vertical value.

This version of the splitting behaves well concerning total energy. No filters or corrections are needed. In addition, the numerical scheme fulfills a discrete maximum principle and a discrete entropy inequality. The gain in terms of computational costs is large, especially in low Froude simulations.

Currently, this work addresses the constant density case; however, ongoing efforts aim to extend the barotropic-baroclinic splitting to variable density scenarios to model situations such as coastal upwelling.

References

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