A curl preserving finite volume scheme by space velocity enrichment. Application to the low Mach number accuracy problem.

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Classical finite volume schemes for compressible Euler system are not accurate on quadrangular mesh at low Mach number in the sense that they do not converge to the incompressible limit when the Mach number tends to zero [2]. The spurious mode that jeopardizes the convergence can be identified and corresponds to the long time limit of a first order wave system whose properties and discretization depend on the scheme used for the compressible system [3]. Then, the low Mach number accuracy can be analysed by studying the long time solution of the associated wave system.

In this presentation, we propose to enrich the velocity space approximation by adding a divergence free element in each cell [5]. Using this new approximation space and a Godunov' numerical flux, the long-time limit of the wave system discretization corresponds to the divergence free component of the initial condition (as expected). This divergence free component will be defined via a discrete Hodge-Helmoltz decomposition.

As a consequence, for Euler equations with *well-prepared* initial and boundary conditions, using this new approximation space and a Roe numerical flux (without any correction), the numerical scheme provides pressure fluctuations of order Mach squared and a divergence of the velocity field of order Mach in the low Mach number limit. In addition, the resulting numerical scheme accurately propagates acoustic waves at low Mach numbers. The theoretical study will be illustrated by numerical results.

This work can be seen as an extension to quadrangles of the properties obtained on triangles [1, 4].

References:

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