An Efficient Entropy Stable Discontinuous Galerkin Spectral Element Method on Heterogeneous Grids

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ABSTRACT

The discontinuous Galerkin spectral element method (DGSEM) has proven to be a highly efficient numerical scheme for the approximation of smooth multiscale problems, e.g., turbulence in the present case. Key aspects for the high efficiency are its excellent parallelization properties and the use of a tensor-product ansatz, where the multidimensional space operator is decomposed into a sequence of one-dimensional operations. Efficient and robust application-oriented large eddy simulations (LES) of turbulence were enabled by the use of entropy-stable DGSEM based on diagonal-norm SBP operators. In its classical tensor-product formulation, the entropy stable DGSEM was first developed for quadrilateral and hexahedral elements. However, for the discretization of complex geometries, more flexible element types, e.g., simplexes, are in general more favorable. In a recent work, Montoya and Zingg [1] presented an efficient entropy stable flux differencing DGSEM formulation on triangular and tetrahedral elements. To benefit from the computational efficiency of the tensor-product ansatz, the authors used a collapsed coordinate transformation on Legendre–Gauss–Lobatto and Legendre–Gauss-Radau nodes, see e.g. [1]. The authors were able to show that their scheme is computationally more efficient as previously developed entropy stable schemes which utilize a full-order basis. Moreover, the intrinsic drawback of small time steps in the collapsed coordinates formulation was resolved by the temporal evolution of modal degrees of freedom (DOFs) instead of nodal ones.

In practical applications such as the flow around an airfoil in the majority of the computational domain, the use of, e.g., hexahedral elements can be favorable and only in a small subset, e.g., simplexes are required. For this, the approach in [1] is extended to triangular prism, pyramid and tetrahedral elements on Legendre–Gauss nodes to enable an efficient entropy stable DGSEM on heterogeneous grids. To avoid any load imbalances due to the additional modal switching of the residual on non-hexahedral elements, a time measurements based partitioning technique is employed for the load balancing.

In the present talk, an entropy stable DGSE scheme on heterogeneous meshes is presented. A particular focus is placed on the collapsed coordinates transformation, the interelement flux computation between different elements types, the modal time-update of the non-hexahedral elements, and the load balancing strategy. The free-stream preservation and the convergence properties are demonstrated based on numerical experiments. We conclude our talk with simulations of the Taylor–Green vortex on a mixed element grid to demonstrate the robustness of the proposed method.

References

[1] T. Montoya and D. W. Zingg, ”Stable and conservative high-order methods on triangular elements using tensor-product summation-by-parts operators”, J. Comp. Phys., 2024.