In this talk, we present reliable a posteriori error estimates for Runge-Kutta discontinuous Galerkin (RKdG) approximations of convection-diffusion systems. We focus on the nonlinear convection-diffusion problems from a hyperbolic point of view, especially on convection-dominated flows and nonlinear advection systems, and ensuring that our estimates remain robust in the vanishing viscosity limit.

A crucial component of our a posteriori analysis is the underlying stability framework of the system. To this end, we develop a general stability framework based on relative entropy stability. In addition, we employ a fully computable space-time reconstruction of the numerical solution that is Lipschitz continuous in space and time. We use the relative entropy framework to bound the difference between the exact solution and the reconstruction in terms of the residual.

We prove that our estimators provide reliable upper bounds for the error of the numerical method. Notably, in the vanishing viscosity limit, these estimators converge to optimal order error bounds for approximations of systems of hyperbolic conservation laws. Finally, we present numerical experiments in order to assess the scaling behavior of the error estimator with respect to the viscosity parameter and with respect to the mesh width.