Hyperbolic relaxation of the Navier-Stokes-Cahn-Hilliard system for incompressible two-phase flows

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In this work, we consider a diffuse-interface model for two incompressible and immiscible fluids at a constant temperature. To investigate the interface dynamics of two-phase flows, we focus on the incompressible Navier-Stokes-Cahn-Hilliard (NSCH) system. The computational cost of the NSCH system is high due to the presence of non-local, non-linear terms and fourth-order derivative term in the Cahn-Hilliard equation. Therefore, we approximate the inviscid part of the NSCH system with a first-order hyperbolic approximate system. We have the following advantages with the first-order hyperbolic approximation: It provides a structure to employ classical numerical methods developed for first-order hyperbolic balance laws. Furthermore, it reduces the computational cost by eliminating the discretization of the fourth-order derivative term and the solution procedure of the Poisson equation for the pressure. The characteristic analysis of the first-order approximate system in one spatial dimension gives four real and distinct eigenvalues. Moreover, as primary results, numerical experiments are performed in one spatial dimension for two purposes. The first purpose is to obtain the wave structure of the first-order hyperbolic approximation. We employ a classical second-order finite volume MUSCL-Hancock scheme for the discretization, where we utilize the *minmod* limiter and a Rusanov flux and solve Riemann problems. The second purpose is to obtain the convergence of the solutions of the first-order hyperbolic aprroximation with the viscosity term, called the *approximate system*, to the NSCH system. Therefore, we implement a conservative second-order finite difference scheme for the spatial discretization and an implicit fifth-order Runge-Kutta method for the time discretization of the approximate system to numerically demonstrate the convergence of the approximate system to the NSCH system for three different test cases: a stationary droplet, the spinodal decomposition and Ostwald ripening.