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Title: Fully-discrete and semi-discrete Lagrangian-Eulerian formulations for hyperbolic systems of conservation laws in three-space dimensions on structured cubical and tetrahedral meshes (*joint work with J. Agudelo, P. Godoi, W. Lambert and J. Pérez*)

Abstract: We present a construction of fully-discrete and semi-discrete Lagrangian-Eulerian schemes for numerically solving three-dimensional scalar (and systems of) hyperbolic conservation laws on structured cubical and tetrahedral meshes. Considering the mathematical challenges of numerics for hyperbolic conservation laws with applications in fluid dynamics, we also briefly present the numerical results in the context of high-performance computing via a MPI environment and its performance using the typical strong scaling metric. We will present several preliminary 3D numerical solutions for systems, for instance, (1) compressible Euler flows with positivity of the density, (2) the classical Orszag-Tang problem in magneto-hydrodynamics, which is well-known to satisfy the notable involution-constrained partial differential equation $\nabla \cdot B = 0$ (this condition is verified numerically by the proposed approach, i.e., without any imposition of an aditional constraint in the formulation), and (3) a nonstrictly hyperbolic three-phase flow system in porous media with a resonance point [1]. To achieve this, we begin by deriving fully-discrete and semi-discrete schemes for a generic three-dimensional hyperbolic conservation law making use of the recent Lagrangian-Eulerian framework [2,3], which is based on the improved concept of no-flow curves/surfaces/manifolds [4,5,7,8,9] to construct a novel approximation for the flux integral that naturally adapt to the conserved quantities in a very simple two-step manner, which is effective for computational implementation: The first Lagrangian evolution step automatically handles the hyperbolic fluxes and, the second step, a Eulerian remap, which allows the use of a single structured cubical and tetrahedral mesh grid, thus eliminating the need for moving meshes while retaining local conservation. Due to the no-flow framework, there is no need to employ/compute the eigenvalues (exact or approximate values) - in fact there is no need to construct the relevant Jacobian of the hyperbolic flux functions, and thus giving rise to an effective weak CFL-stability condition, which is feasible in the computing practice. The method is Riemann-solver-free and, hence, time-consuming field-by-field type decompositions are avoided in the case of multidimensional systems, which retains the same simplicity of the 1D and 2D prior formulations for multidimensional *local* [3,4,6] and *nonlocal* [7] conservation laws as well as for well-balanced Lagrangian-Eulerian methods for shallow water systems [5,8] and nonlinear balance laws [2,9].

Keywords: Three-dimensional hyperbolic systems of conservation laws. Lagrangian-Eulerian method. Fully-discrete and semi-discrete schemes, High Performance Computing. Message Passing Interface MPI.

References:

[1] Lagrangian-Eulerian schemes for hyperbolic conservation laws on cubical and tetrahedral meshes using the MPI parallel implementation, in progress, joint work with J. Agudelo, P. Godoi, W. Lambert and J. Pérez.

[2] A fast, robust, and simple Lagrangian-Eulerian solver for balance laws and applications. Computers & Mathematics with Applications, 77(9) (2019) 2310-2336. https://doi.org/10.1016/j.camwa.2018.12.019 joint work with J. Pérez.

[3] On the conservation properties in multiple scale coupling and simulation for Darcy flow with hyperbolic-transport in complex flows. Multiscale Model. Simul. 18(4) (2020) 1375-1408. https://doi.org/10.1137/20M1320250 joint work with C. Diaz, J. Galvis and J. Pérez.

[4] A triangle-based positive semi-discrete Lagrangian-Eulerian scheme via the weak asymptotic method for scalar equations and systems of hyperbolic conservation laws, Journal of Computational and Applied Mathematics, 437, February 2024, 115465. https://doi.org/10.1016/j.cam.2023.115465 joint work with J. Agudelo and J. Pérez.

[5] A Lagrangian-Eulerian Method on Regular Triangular Grids for Hyperbolic Problems: Error Estimates for the Scalar Case and a Positive Principle for Multidimensional Systems, Journal of Dynamics and Differential Equations, Published: 26 June 2023, Volume 37, pages 749–814, (2025) https://doi.org/10.1007/s10884-023-10283-1 joint work with J. Agudelo, J. Pérez and W. Lambert.

[6] A class of Lagrangian-Eulerian shock-capturing schemes for first-order hyperbolic problems with forcing terms. J. Sci. Comput. 86, (2021) Article number 14. https://doi.org/10.1007/s10915-020-01392-w joint work with V. Matos, J. Pérez and P. Rodríguez-Bermúdez.

[7] Semi-discrete Lagrangian-Eulerian approach based on the weak asymptotic method for nonlocal conservation laws in several dimensions, Journal of Computational and Applied Mathematics, Volume 458 (2025). https://doi.org/10.1016/j.cam.2024.116325 joint work with R. de la Cruz, J. Juajibioy, W. Lambert.

[8] A geometrically intrinsic lagrangian-Eulerian scheme for 2D shallow water equations with variable topography and discontinuous data. Applied Mathematics and Computation, 443 (2023) 127776. https://doi.org/10.1016/j.amc.2022.127776 joint work with E. Bachini, J. Pérez and M. Putti.

[9] An Enhanced Lagrangian-Eulerian Method for a Class of Balance Laws: Numerical Analysis via a Weak Asymptotic Method With Applications, Numerical Methods for Partial Differential Equations, Volume 41, Issue 1 (January 2025; e23163) https://doi.org/10.1002/num.23163 joint work with E. Pandini and W. Lambert.