A first-order hyperbolic reformulation of the Cahn-Hilliard equation

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ABSTRACT

I will present a new first-order hyperbolic reformulation of the Cahn-Hilliard equation. The model is obtained from the combination of augmented Lagrangian techniques, with a classical Cattaneo-type relaxation that allows to reformulate diffusion equations as augmented first order hyperbolic systems with stiff relaxation source terms. The proposed system is proven to be hyperbolic and to admit a Lyapunov functional, in accordance with the original equations. A new numerical scheme is proposed to solve the original Cahn-Hilliard equations based on conservative semi-implicit finite differences, while the hyperbolic system was numerically solved by means of a second order MUSCL-Hancock finite volume scheme. The proposed approach is validated through a set of classical benchmarks such as spinodal decomposition, Ostwald ripening and exact stationary solutions.

References

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