Title: Uncertainty quantification for hyperbolic PDEs with many uncertain parameters

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Abstract:

The present work concerns the uncertainty quantification (UQ) of nonlinear hyperbolic systems with many uncertain parameters. The aim is to quantify the dependency of the solution to the initial condition uncertainty. The basis for our work is the stochastic finite volume method (SFV), which allows us to rewrite the UQ problem as a deterministic problem with new variables [1].

In the case of many uncertain parameters, the SFV is combined with low-rank tensors, namely tensor trains [2], to keep the computational cost reasonable. We propose here a new way of combining the SFV and the tensor trains: the physical space and time dimensions are kept as full tensors, while all stochastic dimensions are compressed together into a tensor train. This hybrid format makes it straightforward to re-use existing finite volume algorithms for UQ problems.

We present numerical experiments using an adapted MUSCL scheme in 1D. The Burgers' equation and the Euler equations are solved with uncertain initial conditions. The hybrid format requires much less parameters than a full SFV would, and is able to capture nonlinear features such as shock waves.

References:

[1] R. Abgrall and S. Tokareva. *The stochastic finite volume method*, in Uncertainty Quantification for Hyperbolic and Kinetic Equations, vol. 14, pp. 1–57

[2] I. Oseledets, *Tensor-train decomposition*, SIAM J. Sci. Comput. vol. 33, pp. 2295–2317

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