On well-balanced finite difference, finite volume and discontinuous Galerkin schemes for the Einstein-Euler system of general relativity

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(joint work with Olindo Zanotti, Ilya Peshkov, Elena Gaburro, Gabriella Puppo)

In this talk we present a new family of well-balanced conservative CWENO finite difference schemes and discontinuous Galerkin (DG) finite element schemes with subcell finite volume (FV) limiter for the numerical solution of the Einstein–Euler equations of general relativity based on a first order hyperbolic reformulation of the Z4 formalism. The first order Einstein-Euler Z4 system, which is composed of 64 equations, is analysed and proven to be strongly hyperbolic for a general metric. The well-balancing is achieved for arbitrary but *a priori* known equilibria by subtracting a discrete version of the equilibrium solution from the discretized time-dependent PDE system. Special care has also been taken in the design of the numerical viscosity so that the well-balancing property is achieved. As for the treatment of low density matter, e.g. when simulating massive compact objects like neutron stars surrounded by vacuum, we have introduced a new filter in the conversion from the conserved to the primitive variables, preventing superluminal velocities when the density drops below a certain threshold, and being potentially also very useful for the numerical investigation of highly rarefied relativistic astrophysical flows.

We furthermore present a novel family of central WENO finite difference schemes for a new first order reformulation of the classical BSSNOK system.

Thanks to these improvements, all standard tests of numerical relativity are successfully reproduced, reaching four main achievements: (i) we are able to obtain stable long term simulations of stationary black holes, including Kerr black holes with extreme spin, which after an initial perturbation return perfectly back to the equilibrium solution up to machine precision; (ii) a (standard) TOV star under perturbation is evolved in pure vacuum ($\rho = p = 0$) up to $t = 1000$ with no need to introduce any artificial atmosphere around the star; and, (iii) we solve the head on collision of two punctures black holes, that was previously considered un–tractable within the FO-Z4 formalism, (iv) we perform a stable long-time evolution of a rotating binary black hole merger based on the new CWENO schemes for first order reformulation of the BSSNOK system.

References

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