Title: Numerical solutions of the Hunter–Saxton equation

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Abstract: Solutions of the Hunter–Saxton equation might enjoy wave breaking in finite time. This means that classical solutions in general do not exist globally, but only locally in time since their spatial derivative might become unbounded from below pointwise in finite time, while the solution itself remains bounded. In addition, energy concentrates on sets of measure zero when wave breaking occurs. As a consequence the prolongation of solutions beyond wave breaking is non-unique.

We will present a numerical method for α-dissipative solutions, i.e., solutions where the energy is manipulated at breaking time by taking out an α-part of the concentrated energy. This method combines a carefully chosen projection operator with a generalized method of characteristics and an iteration scheme, which enforces minimal time steps whenever wave breaking times cluster. Convergence is obtained for any admissible initial data, while a convergence rate is derived under some additional constraints on the initial data.

References:

T. Christiansen and K. Grunert, A numerical view on α-dissipative solutions of the Hunter–Saxton equation, ESAIM Math. Model. Numer. Anal. 59 579–612 (2025).

T. Christiansen and K. Grunert, Rate of convergence for numerical α-dissipative solutions of the Hunter–Saxton equation, arXiv:2411.07712.

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