A novel structure-preserving numerical method to solve random hyperbolic systems of conservation laws is presented. The method uses a concept of generalized, measure-valued solutions to random conservation laws. {This yields a linear partial differential equation with respect to the Young measure and allows to compute the approximation based on linear programming problems. We analyse structure-preserving properties of the derived numerical method and discuss its advantages and disadvantages. Numerical results for one-dimensional Burgers equation and the isentropic Euler equations and comparisons with stochastic collocation method illustrate the behavior of the proposed numerical method.