Recent progress on structure-preserving numerical methods for dispersive wave equations

Joshua Lampert[*1], Hendrik Ranocha[†2], and Mario Ricchiuto[‡3]

[1]Department of Mathematics, University of Hamburg, Germany

[2]Institute of Mathematics, Johannes Gutenberg University Mainz, Germany

[3]Inria centre at the University of Bordeaux, France

Water wave propagation problems can often be modeled using a depth-averaged shallow water approximation, e.g., tsunami propagation or dam breaks. The classical first-order hyperbolic shallow water equations are sufficient to describe the wave dynamics in many cases. However, some applications require more accurate models, e.g., nonlinear dispersive wave equations taking higher-order effects into account. Several variants of such models exist and are used in practice, e.g., the BBM-BBM equations, the Serre-Green-Naghdi system, and hyperbolic approximations thereof. In this talk, we will review some recent developments of structure-preserving numerical methods. In particular, we will consider invariants such as the total energy and study efficient numerical methods yielding qualitative and quantitative improvements compared to standard schemes. To develop structure-preserving schemes, we make use of the general framework of summation-by-parts (SBP) operators in space, unifying the analysis of finite difference, finite volume, finite element, discontinuous Galerkin, and spectral methods. Finally, we combine structure-preserving spatial discretizations with relaxation methods in time to obtain fully-discrete, energy-conservative schemes. This talk is based on [1, 2]

References

[1] J. Lampert and H. Ranocha. *Structure-Preserving Numerical Methods for Two Nonlinear Systems of Dispersive Wave Equations*. Feb. 2024. arXiv: 2402.16669 [math.NA].

[2] H. Ranocha and M. Ricchiuto. *Structure-preserving approximations of the Serre-Green-Naghdi equations in standard and hyperbolic form*. Aug. 2024. arXiv: 2408.02665 [math.NA].

[*]ORCID: 0009-0007-0971-6709

[†]ORCID: 0000-0002-3456-2277

[‡]ORCID: 0000-0002-1679-7339