Numerical Approximation of Multispecies Kinematic Flow Models through Positivity Preserving WENO Schemes

Juan Barajas-Calonge*, Raimund Bürger†, Pep Mulet‡ and Luis M. Villada*

* GIMNAP-Departamento de Matemática
Universidad del Bío-Bío, Avenida Collao 1202, Concepción, Chile
e-mail: juan.barajas2001@alumnos.ubiobio.cl

† CI^{2}MA and Departamento de Ingeniería Matemática
Universidad de Concepción, Casilla 160-C, Concepción, Chile

‡ Departament de Matemàtiques
Universitat de València, Av. Vicent Andrés Estellés s/n, Burjassot, Spain

ABSTRACT

Multispecies kinematic flow models are defined by systems of N strongly coupled, nonlinear first-order conservation laws, where the solution is a vector of N partial volume fractions or densities. The solution vector should take values in a set of physically relevant values (i.e., the components are nonnegative and sum up at most to a given maximum value). In the 1D case, it is shown that this set, the so-called invariant region, is preserved by numerical solutions produced by a new family of high-order finite volume numerical schemes adapted to this class of models [1]. To achieve this property, and motivated by [3], a pair of linear scaling limiters is applied to a high-order central weighted essentially non-oscillatory (CWENO) polynomial reconstruction [2] to obtain invariant-region-preserving (IRP) high-order polynomial reconstructions. These reconstructions are combined with a first order numerical flux to obtain a high-order numerical scheme for the system of conservation laws. It is proved that this scheme satisfies an IRP property under a suitable CFL condition. For the 2D case, we study a polydisperse sedimentation model consisting in a system of conservation laws coupled with a Stokes problem describing the velocity of the mixture. We propose a second order positivity preserving FV-CWENO scheme for the numerical approximation. The theoretical analysis is corroborated with numerical simulations in some scenarios of interest.

REFERENCES

[1] J. Barajas-Calonge, R. Bürger, P. Mulet and L.M. Villada, Invariant-region-preserving central WENO schemes for one-dimensional multispecies kinematic flow models, Preprint 2024-26, CI^{2}MA, Universidad de Concepción.

[2] D. Levy, G. Puppo G. Russo, Central WENO schemes for hyperbolic systems of conservation laws, ESAIM: Mathematical Modelling and Numerical Analysis. Vol. 33, pp. 547-571, 1999.

[3] X. Zhang, C.-W. Shu, On maximum-principle-satisfying high order schemes for scalar conservation laws, J. Comput. Phys. Vol. 229, 3091-3120, 2010.