Numerical staggered conservative scheme for the simulation of low Mach number flows

Esteban COIFFIER, LMAP/Cea Saclay STMF - France, Pau / Saclay
Jonathan JUNG, LMAP/Inria CAGIRE - France, Pau
Michael NDJINGA, Cea Saclay STMF - France, Saclay
Vincent PERRIER, LMAP/Inria CAGIRE - France, Pau

We are interested here in the simulation of compressible fluid mechanics equations in a low Mach number regime. More specifically, we study the numerical approximation of the barotropic Euler equations using finite volume/finite element methods.
Low Mach number flows are notoriously difficult to simulate with classical finite volume methods, mainly because their accuracy depends on the mesh shape [2]. Inspired by the MAC scheme [3] (introduced for the simulation of incompressible fluids), one of the proposed solutions to address this issue consists of staggering the velocity degrees of freedom at the mesh faces to improve the approximation of the divergence operator. The challenge of such a placement of unknowns lies in defining conservation, compared to colocated finite volume methods where it directly results from the scheme's formulation.
In [4], the authors proposed conservative staggered schemes based on Crouzeix-Raviart and Rannacher-Turek finite elements for each velocity component.
Our approach follows this line of research with the following originality : we introduce a staggered discretization based on the de Rham complex of Nédélec-Raviart-Thomas finite elements [1]. More precisely, the velocity is in the Raviart-Thomas space, requiring only one degree of freedom per mesh face in any spatial dimension.
The interest in relying on a discrete de Rham complex is illustrated through an asymptotic analysis in the Mach number [5] :

i) The complex allows us to demonstrate the existence of a discrete Hodge decomposition, which helps identify the low Mach limit of the scheme.
ii) Using this formalism, stabilization terms have been constructed to propagate low Mach number acoustic waves in explicit time integration.

In this presentation, we will introduce both the theoretical tools that ensure accuracy at low Mach numbers and the procedure for obtaining a conservative finite volume scheme. We will illustrate the scheme's properties through numerical simulations in 2d.

[1] A. Ern, J.-L. Guermond. *Theory and practice of finite elements*, vol. 159. Springer, 2004.

[2] H. Guillard. *On the behavior of upwind schemes in the low mach number limit. iv : P0 approximation on triangular and tetrahedral cells.* Computers & fluids, **38(10)**, 1969–1972, 2009.

[3] F. H. Harlow. *Mac numerical calculation of time-dependent viscous incompressible flow of fluid with free surface.* Phys. Fluid, **8**, 12, 1965.

[4] R. Herbin, W. Kheriji, J.-C. Latché. *On some implicit and semi-implicit staggered schemes for the shallow water and euler equations.* ESAIM : Mathematical Modelling and Numerical Analysis, **48(6)**, 1807–1857, 2014.

[5] J. Jung, V. Perrier. *Steady low mach number flows : identification of the spurious mode and filtering method.* Journal of Computational Physics, **468**, 111462, 2022.

Contact : esteban.coiffier@cea.fr