POD reduction for finite volume schemes applied to nonlinear hyperbolic PDE systems

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We introduce a reduced-order modeling strategy for systems of nonlinear hyperbolic partial differential equations, based on proper orthogonal decomposition (POD) techniques. Our aim is to construct reduced models that preserve essential mathematical structures of the full-order system, including the well-balanced property.

In many applications, especially in geophysical fluid dynamics, capturing steady states exactly and ensuring a correct balance between fluxes and source terms is essential. To address this, we design a well-balanced POD-Galerkin approach. The methodology begins with the generation of high-fidelity snapshots using a finite volume solver equipped with a well-balanced scheme. These snapshots are then projected onto a tailored POD basis constructed to retain key stationary solutions. The resulting reduced model satisfies a discrete well-balanced property by design.

We apply this methodology, among others, to the one-dimensional shallow water equations with variable topography, a prototypical hyperbolic system with nontrivial steady states. Numerical experiments demonstrate that the reduced model reproduces the dynamics of the full system with high fidelity, while significantly reducing computational cost. Crucially, the preservation of steady states at the reduced level allows for robust and physically consistent simulations.

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