High order schemes on Voronoi-like tessellations: space-time control volumes around topology changes and multidimensional Riemann solvers

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ABSTRACT

The framework of this work is that of high order Finite Volume and Discontinuous Galerkin methods for solving hyperbolic equations on unstructured polygonal and polyhedral tessellations constructed as centroid-based dual of Delaunay meshes.

First, I will concentrate on the treatment of topology changes in the case of direct Arbitrary-Lagrangian-Eulerian (ALE) schemes on Voronoi meshes which are regenerated at each time-step and that should be connected via space-time control volumes[5, 4]. Unfortunately, just linking the existing elements, by collapsing or expanding their edges, leaves a *hole* in the space-time. Thus, we fill this hole by considering an additional *space-time degenerate hole-like element* that we call *sliver* element.

Here, I will describe the shape and the numerical treatment of a sliver element in the 2D+time framework with a direct ALE ADER Discontinuous Galerkin method. Then, I will justify the *stability* of our approach applied to sliver elements by presenting a von Neumann stability analysis performed in the 1D+time setting at the aid of a fictitious 1D+time sliver element [2]. Finally, I will show preliminary novel results about sliver elements in 3D+time: their degenerate *4D hole-like* shape will be fully characterized and various visualization strategy will be proposed [1].

Furthermore, I will present two novel complete genuinely multidimensional Riemann solvers [3] that benefit from the Voronoi tessellations and are extended to the high order of accuracy thanks to a CWENO-ADER algorithm. The first is a direct extension of the Osher-Solomon Riemann solver via integration of the dissipation term over manifolds and ii) the second is based on the so-called N-scheme, an optimal upwind flux originally developed for residual distribution methods.

References

[1] M. Klima, E. Gaburro. “High order treatment of topology changes on 3D Voronoi-like tessellation via 4D hole-like sliver elements.”, *in preparation*, 2025.

[2] M. Bonafini, E. Gaburro, D. Torlo “Stability analysis for explicit and implicit ADER Discontinous Galerkin methods in the Eulerian and Lagrangian framework including degenerate Italia elements”, *in preparation*, 2025.

[3] E. Gaburro, M. Ricchiuto, M. Dumbser. “On general and complete multidimensional Riemann solvers for nonlinear systems of hyperbolic conservation laws.”, *submitted*, 2025.

[4] E. Gaburro. “A Unified Framework for the Solution of Hyperbolic PDE Systems Using High Order Direct Arbitrary-Lagrangian–Eulerian Schemes on Moving Unstructured Meshes with Topology Change”. *Archives of Computational Methods in Engineering*, 2021.

[5] E. Gaburro, W. Boscheri, S. Chiocchetti, C. Klingenberg, V. Springel, M. Dumbser. “High order direct Arbitrary-Lagrangian-Eulerian schemes on moving Voronoi meshes with topology changes”. *Journal of Computational Physics*, 2020.

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