A GRP-based compact reconstruction for finite volume schemes applied to hyperbolic conservation laws

Lucas O. Müller[1], Gino I. Montecinos[2] & Eleuterio F. Toro[3]

1. Department of Mathematics, University of Trento, Italy
2. Department of Mathematical Engineering, Universidad de la Frontera, Chile
3. Laboratory of Applied Mathematics, DICAM, University of Trento, Italy

Abstract

Reconstruction procedures are one of the means to build numerical schemes of high-order of accuracy for hyperbolic partial differential equations. To obtain reconstruction polynomials it is required the use of a stencil, a set of cell averages for finite volume methods. The size of stencils is related to accuracy, it increases when the accuracy does so.

The computation of reconstruction stencils can become cumbersome when dealing with three-dimensional problems and unstructured meshes. Furthermore, there are one-dimensional (1D) problems where reconstruction stencils pose problems, specifically those in which domains are small, making the role of boundaries extremely important. A relevant example is the one given by 1D hyperbolic systems acting on networks, or when 1D domains are coupled to models consisting of ordinary differential equations. Preserving the order of accuracy of the numerical scheme employed in 1D domains introduces significant complexity to the overall numerical approach.

For these reasons, the ability to produce reconstruction polynomials with a minimal number of elements in stencils is very attractive. A notorious and successful example of compact-stencil schemes is that of discontinuous Galerkin (DG) schemes. These schemes avoid the use of reconstruction polynomials by directly evolving local polynomials in each computational cell, while interacting only with neighbouring cells. This results in much more compact stencils with respect to the ones employed with finite volume schemes. However, DG schemes have stability requirements that reduce the CFL coefficient for increasing order of accuracy.

In this work, we present a reconstruction procedure for finite volume schemes that can reach up to fifth order of accuracy, while maintaining a DG-like stencil. In order to achieve this, reconstruction is performed using neighbouring cell data and the solution of Generalized Riemann Problems (GRP) at cell interfaces. The procedure does not pose restrictions to CFL coefficients with respect to those use with classical finite volume schemes and can be formulated in both linear and nonlinear fashions. The methodology is particularly appropriate for GRP-based solvers such as [1, 2].

We assess the proposed method with the linear advection equation and two nonlinear systems, i.e. the 1D blood flow equations and the Euler equations. We provide numerical evidence of order of accuracy, stability and efficiency. The results are compared with those obtained using well-known reconstruction approaches [3], as well as with the fully discrete DG scheme proposed in [4]. Our findings, limited for now to the one-dimensional case, are promising in terms of accuracy and efficiency.

Keywords: Reconstruction procedure, Generalized Riemann Problems, Godunov theorem

References

[1] E. F. Toro and V. A. Titarev. Solution of the generalised Riemann problem for advection–reaction equations. *Proceedings of the Royal Society of London A*, 458:271–281, 2002.

[2] M. Dumbser, C. Enaux, and E. F. Toro. Finite volume schemes of very high order of accuracy for stiff hyperbolic balance laws. *Journal of Computational Physics*, 227(8):3971–4001, 2008.

[3] M. Dumbser and M. Käser. Arbitrary high order non-oscillatory finite volume schemes on unstructured meshes for linear hyperbolic systems. *Journal of Computational Physics*, 221(2):693–723, 2007.

[4] M. Dumbser, D. Balsara, E. F. Toro, and C. D. Munz. A unified framework for the construction of one-step finite volume and discontinuous Galerkin schemes on unstructured meshes. *Journal of Computational Physics*, 227:8209–8253, 2008.