Advances in Numerical Methods for Dispersive Models in Geophysical Flow Applications

C. Escalante-Sánchez

Departamento de Matemática Aplicada, Universidad de Málaga, Spain.

ABSTRACT

This work presents a general framework for the numerical simulation of dispersive geophysical flows, based on the multilayer-moment technique [1,2], with an emphasis on its applicability to realistic scenarios. The resulting methodology leads to non-hydrostatic pressure models that involve only first-order derivatives, while achieving arbitrarily high accuracy in the dispersion relation. Several robust numerical schemes are discussed, with a particular focus on projection-correction methods [3,4,5] that enforce incompressibility, as well as on recent strategies that relax this constraint [6,7,8]. The numerical challenges and advances associated with the relaxation approach applied to the more general multilayer system are also addressed.

Keywords: dispersive models, non-hydrostatic pressure, multilayer methods, projection-correction schemes, hyperbolic relaxation.

This research has been partially funded by MCIN/AEI/10.13039/501100011033 and by "ERDF A way of making Europe," by the European Union through the Grant PID2022-137637NB-C21.

References

1. Fernández-Nieto, E., Parisot, M., Penel, Y., & Sainte-Marie, J. (2018). A hierarchy of dispersive layer-averaged approximations of Euler equations for free surface flows. Communications in Mathematical Sciences, 16(5), 1169–1202.

2. Escalante, C., Fernández-Nieto, E., Garres-Díaz, J., Luna, T., & Penel, Y. (2023). Non-hydrostatic layer-averaged approximation of Euler system with enhanced dispersion properties. Computational and Applied Mathematics, 42(4).

3. Escalante, C., Luna, T., & Castro, M. (2018). Non-hydrostatic pressure shallow flows: GPU implementation using finite volume and finite difference scheme. Applied Mathematics and Computation, 338, 631–659.

4. Escalante, C., Fernández-Nieto, E., Garres-Díaz, J., Luna, T., & Penel, Y. (2023). Non-hydrostatic layer-averaged approximation of Euler system with enhanced dispersion properties. Computational and Applied Mathematics, 42(4).

5. Escalante, C., Luna, T., Cantero-Chinchilla, F., & Castro-Orgaz, O. (2024). Vertically averaged and moment equations: New derivation, efficient numerical solution and comparison with other physical approximations for modeling non-hydrostatic free surface flows. Journal of Computational Physics, 504, 112882.

6. Escalante, C., Dumbser, M., & Castro, M. (2019). An efficient hyperbolic relaxation system for dispersive non-hydrostatic water waves and its solution with high order discontinuous Galerkin schemes. Journal of Computational Physics, 394, 385–416.

7. Busto, S., Dumbser, M., Escalante, C., Favrie, N., & Gavrilyuk, S. (2021). On High Order ADER Discontinuous Galerkin Schemes for First Order Hyperbolic Reformulations of Nonlinear Dispersive Systems. Journal of Scientific Computing, 87(2).

8. Escalante, C., & Luna, T. (2020). A General Non-hydrostatic Hyperbolic Formulation for Boussinesq Dispersive Shallow Flows and Its Numerical Approximation. Journal of Scientific Computing, 83(3).