A posteriori error estimates for high order approximations of hyperbolic conservation laws

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We prove rigorous a-posteriori error estimates for high order Runge-Kutta discontinuous Galerkin (RKDG) schemes for nonlinear systems of hyperbolic conservation laws in one spatial dimension. The estimates are based on recent results for first-order finite volume schemes [1]. Our estimators rely on recent stability results by Bressan, Chiri and Shen [2], a new way to localize residuals and a novel method to compute dual norms of these local residuals. Computing dual norms becomes possible by a reconstruction of the RKDG solution continuously within space-time cells and a direct approximation of the localized residual.

References

[1] J. Giesselmann and A. Sikstel. A-posteriori error estimates for systems of hyperbolic conservation laws via computing negative norms of local residuals. *IMA Journal of Numerical Analysis*, page drae111, 03 2025.

[2] A. Bressan, M. T. Chiri, and W. Shen. A posteriori error estimates for numerical solutions to hyperbolic conservation laws. *Archive for Rational Mechanics and Analysis*, 241(1):357–402, July 2021.