Numerical methods for hyperbolic problems in (in)compressible multiphase flow

Christian Rohde
University of Stuttgart, Germany

Abstract:

Diffuse-interface and phase-field models are frequently used to describe the multiphase dynamics of compressible or incompressible fluids. Examples include Navier-Stokes-Korteweg systems and Navier-Stokes-Cahn-Hilliard or Navier-Stokes-Allen-Cahn systems.

The solutions of such systems exhibit strong non-local effects not only due to the second-order viscosity operators but also due to higher-order capillarity operators, and velocity divergence constraints (in the incompressible case).

As a consequence the numerical approximation of convection-dominated regimes or flow with high variations in density becomes intricate.

We suggest a number of first-order approximative systems. They rely on the combination of artificial-compressibility approaches, relaxation approximations and friction-type approximations for the capillarity operators. It is shown that the systems are equipped with entropy-entropy flux pairs with convex (mathematical) entropies which renders them to be hyperbolic. Similar to low Mach number regimes in compressible flow the approximations involve fast characteristic waves. We suggest tailored Finite-Volume and Discontinuous-Galerkin methods from the field of hyperbolic conservation laws which allow to address fluid regimes which are hardly accessible by standard discretizations of original models.