## A new compatible semi-implicit schemes for the Maxwell-Munz system.

M. Dumbser[1], **A. Lucca**[1], I. Peshkov[1], O. Zanotti[1]

[1] Department of Civil, Environmental and Mechanical Engineering, University of Trento, Italy

In the proposed talk we will consider the Maxwell-GLM system, which augments the original vacuum Maxwell equations via a generalized Lagrangian multiplier approach (GLM) by adding two supplementary acoustic subsystems. It was originally introduced by Munz *et al.* [1] for purely numerical purposes in order to treat the divergence constraints of the magnetic and electric fields in the vacuum Maxwell equations. The mathematical structure of the augmented Maxwell-GLM system is very intriguing and contains an interesting combination of curl-curl and div-grad operators, which is usually not present in classical PDE systems of continuum physics. We have shown that it can be derived from an underlying variational principle, has a symmetric hyperbolic structure and admits an extra conservation law for the total energy density. Therefore, the Maxwell-GLM system extends the class of symmetric hyperbolic and thermodynamically compatible (SHTC) systems established by Godunov and Romenski. The system is by itself very interesting from a mathematical point of view and can thus serve as an useful prototype system for the development of new structure-preserving numerical methods. In this direction, we will present a new exactly energy-conserving and asymptotic-preserving finite volume scheme for the Maxwell-GLM system. The method introduced is a vertex-based staggered semi-implicit scheme that preserves the fundamental vector calculus identities $\nabla \cdot \nabla \times \mathbf{A} = 0$ and $\nabla \times \nabla \phi = 0$ exactly on the discrete level thanks to the use of mimetic discrete differential operators and an appropriate location of the variables on the grid. Furthermore, the structure-preserving staggered semi-implicit scheme is exactly compatible with the divergence free condition of the electric and magnetic fields and is asymptotic-preserving. The discrete divergence of the magnetic and electric fields converges quadratically to zero consistently with the limit $\epsilon = c_0/c_h \to 0$, where $c_0 \in \mathbb{R}^+$ is the vacuum light speed of the original Maxwell equations and $c_h \in \mathbb{R}^+$ is the speed associated with the two acoustic subsystems. The last property of the presented scheme is that it exactly preserves the global discrete total energy in the case of periodic boundaries. Since the scheme proposed in this talk is able to satisfy all the properties mentioned, it goes beyond the set of known numerical methods for the Maxwell-GLM system.

### REFERENCES

[1] C. Munz, P. Omnes, R. Schneider, E. Sonnendrücker, and U. Voss. Divergence Correction Techniques for Maxwell Solvers Based on a Hyperbolic Model. *Journal of Computational Physics*, 161, 484–511, 2000.